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A New Predictive Ratio Estimator

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SUMMARY

Using the predictive approach advocated by Basu [1] we develop an almost unbiased ratio estimator of a finite population mean which is found to be more efficient than its competitors.

Key words : Almost unbiased ratio estimator, efficiency, mean square error, predictive approach.

Introduction and construction of the new predictive ratio estimator

Let y_i and x_i $(1 \leq i \leq N)$ be the values of two positively correlated variates y and x defined on a finite population of N units with means $\overline{Y}$ and $\overline{X}$ respectively. Let $(\overline{y}, \overline{x})$ and $(\overline{Y}_r, \overline{X}_r)$ denote respectively the means over a simple random without replacement samples of n units and the unsampled residuum. Under the usual prediction approach of Basu [1] an estimator of $\overline{Y}$ is given by

$$\hat{\overline{Y}} = \frac{n}{N}\overline{y} + \frac{N-n}{N}T$$

where T is the implied predictor of $\overline{Y}_r$. If we use $T_R = \overline{y}\,\overline{X}_r/\overline{x}$ as a predictor of $\overline{Y}_r$, $\hat{\overline{Y}}$ reduces to the classical ratio estimator $\hat{\overline{Y}}_R = \overline{y}\,\overline{X}/\overline{x}$ (cf. Srivastava [3]).

Using Taylor linearization method (e.g. Cochran [2]) and noting that to $O(n^{-1})$,

$$E(T_R) = \overline{Y}\left[1 + \frac{N}{N-n}\left(\frac{V(\overline{x})}{\overline{X}^2} - \frac{Cov(\overline{x}, \overline{y})}{\overline{X}\,\overline{Y}}\right)\right]$$

an almost unbiased ratio estimator of $\overline{Y}$ can be obtained as

$$T_{MR} = T_R\left[1 + \theta\frac{N}{N-n}(c_{xy} - c_x^2)\right],$$

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$$\mu_{002}(r, x, y) = 18.71, \ \mu_{020}(r, x, y) = 12.50, \ \mu_{200}(r, x, y) = 0.16$$

$$\mu_{210}(r, x, y) = -0.10, \ \mu_{220}(r, x, y) = 1.90, \ \mu_{111}(r, x, y) = 1.08$$

Computations were done for n = 3. The table 4.2 shows the variance and RE of Y_u over other four estimators considered in the study. The variances or mse's were computed as mentioned for population I. The maximum gain in efficiency over all the estimators is observed when θ assumes the optimum value of θ = – 1.00. The estimator Y_u is seen to be more efficient as compared to $\overline{Y}_{HR}$ even when θ departs in the neighbourhood of its optimum, the range being –0.6 to –1.38. The common range of θ values in which the estimator Y_u is superior to both $\bar{y}_r$ and $\bar{y}_r'$ is comparatively smaller ranging from –0.77 to –1.22. It is to be noted that the estimator $\hat{Y}_6$ a member of $\hat{Y}_u$ which coincides with the optimum estimator, recorded maximum gain in efficiency as compared to all other estimators.

Empirical study thus demonstrates that the survey practitioner can generate estimators which are more efficient than sample mean as well as ratio and Hartley and Ross [1] estimators from the proposed class of estimators Y_u for certain optimum or near optimum values of θ. It is possible in practice to obtain a near optimum value of θ using (3.6) or (3.10) by substituting the values of estimated variances and covariances from the data at hand.

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REFERENCES

[1] Hartley, H.O. and Ross, A, 1954. Unbiased ratio estimators, *Nature*, **174**, 270–271.

[2] Singh, D., Singh, P. and Kumar, P., 1976. Hand book on sampling methods, Indian Agricultural Statistics Research Institute (ICAR), New Delhi.

[3] Srivenkataramana, T. and Tracy, D.S., 1980. An alternative to ratio method in sample surveys. *Ann. Inst. Statist. Math.*, **32**, part A, 111–120.

[4] Sukhatme, P.V., Sukhatme, B.V., Sukhatme, S. and Ashok, C., 1984. Sampling theory of surveys with application. Third edition, Iowa State University Press. Ames, Iowa (USA) and Indian Society of Agricultural Statistics, New Delhi, India.

which is analogous to the almost unbiased ratio estimator

$$\hat{Y}_T = \hat{Y}_R\left[1 + \theta\,(c_{xy} - c_x^2)\right]$$

defined by Tin [4], where $\theta = n^{-1} - N^{-1}$, $c_{xy} = \sum_{i \in s} (x_i - \bar{x})\,(y_i - \bar{y})/(n-1)\,\bar{x}\,\bar{y}$ and $c_x^2 = \sum_{i \in s} (x_i - \bar{x})^2/(n-1)\,\bar{x}^2$. Substituting T_{MR} and T in the predictive equation (1.1), we now define a new almost unbiased predictive ratio estimator

$$\hat{Y}_{MR} = \hat{Y}_R + \theta\, T_R\,(c_{xy} - c_x^2).$$

2. *Comparison of* $\hat{Y}_{MR}$, $\hat{Y}_T$ and $\hat{Y}_R$

To terms of order n^{-2}, Tin ([4]) obtained

$$\text{MSE}\,(\hat{Y}_T) = \bar{Y}^2\Big[\theta(C_{02} - 2C_{11} + C_{20}) + 2\,\frac{\theta}{N}(C_{12} - 2C_{21} + C_{30}) + \theta^2\Big\{2(C_{20} - C_{11})^2 + (C_{20}\,C_{02} - C_{11}^2)\Big\}\Big], \quad (2.1)$$

where $C_{pq} = K_{pq}/\bar{X}^p\,\bar{Y}^q$, K_{pq} being the (p, q)th cumulant of x and y. The author also made a comparison between $\hat{Y}_T$ and $\hat{Y}_R$ and showed that to $O(n^{-2})$

$$\text{MSE}\,(\hat{Y}_T) < \text{MSE}\,(\hat{Y}_R) \quad (2.2)$$

Using the same notations and approximations used by Tin ([4]), to $O(n^{-2})$ we may obtain

$$\text{MSE}\,(\hat{Y}_{MR}) = \bar{Y}^2\Big[\theta(C_{02} - 2C_{11} + C_{20}) + 2\,\frac{\theta}{N}(C_{12} - 2C_{21} + C_{30}) + \theta^2\Big\{2\,\frac{N-2n}{N-n}\,(C_{20} - C_{11})^2 + (C_{20}\,C_{02} - C_{11}^2)\Big\}\Big], \quad (2.3)$$

Comparing (2.3) with (2.1) we have

$$\text{MSE}\,(\hat{Y}_{MR}) < \text{MSE}\,(\hat{Y}_T) \quad (2.4)$$

and then combining the results in (2.2) and (2.4) we get

$$\text{MSE}\,(\hat{Y}_{MR}) < \text{MSE}\,(\hat{Y}_T) < \text{MSE}\,(\hat{Y}_R) \quad (2.5)$$

Thus, the new estimator $\hat{Y}_{MR}$ seems preferable to its competitors; i.e., $\hat{Y}_R$ and $\hat{Y}_T$ on grounds of bias and mean square error.

REFERENCES

[1] Basu, D., 1971. An essay on the logical foundations of survey sampling, Part I. *Foundations of Statistical Inference*, Ed. by V.P. Godambe and D.A. Sprott, New York, 203–233.

[2] Cochran, W.G., 1977. *Sampling Techniques* (Third Edition). Wiley Eastern Limited.

[3] Srivastava, S.K., 1983. Predictive estimation of finite population mean using product estimator. *Metrika*, **30**, 93–99.

[4] Tin, M., 1965. Comparison of some ratio estimators. *J. Amer. Statist. Assoc.*, **60**, 294–307.